Deep generative modeling: Implicit models

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Deep Learning

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TYPES OF GENERATIVE MODELS

Generative model

- **Autoregressive (*e.g.*, PixelCNN)**
- **Flow-based (e.g., RealNVP, GLOW)**
- **Latent variable models**
  - **Implicit models (*e.g.*, GANs)**
  - **Prescribed models (*e.g.*, VAE)**

GENERATIVE MODELS

	Training	Likelihood	Sampling	Compression
Autoregressive models (e.g., PixelCNN)	Stable	Exact	Slow	No
Flow-based models (e.g., RealNVP)	Stable	Exact	Fast/Slow	No
Implicit models (e.g., GANs)	Unstable	No	Fast	No
Prescribed models (e.g., VAEs)	Stable	Approximate	Fast	Yes

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DENSITY NETWORKS

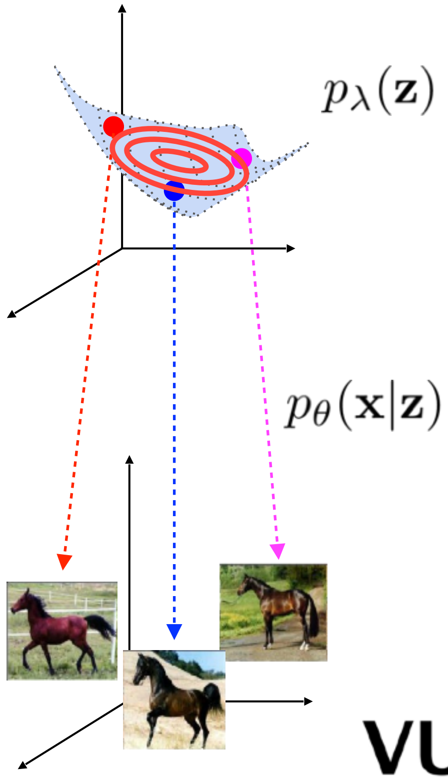
Generative process:

$$1.\ \mathbf{z} \sim p_{\lambda}(\mathbf{z})$$

$$2.\ \mathbf{x} \sim p_{\theta}(\mathbf{x} \mid \mathbf{z})$$

The log-likelihood function:

$$\log p(\mathbf{x}) = \log \int p_{\theta}(\mathbf{x} \mid \mathbf{z}) p_{\lambda}(\mathbf{z}) \mathrm{d}\mathbf{z}$$



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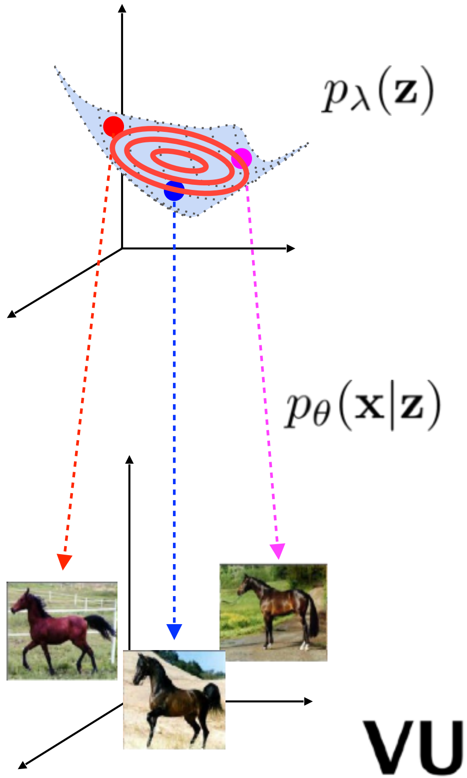
$$\approx \log \frac{1}{S} \sum_{s=1}^{S} \exp\left(\log p_\theta\left(\mathbf{x} \mid \mathbf{z}_s\right)\right)$$

It could be estimated by MC samples.



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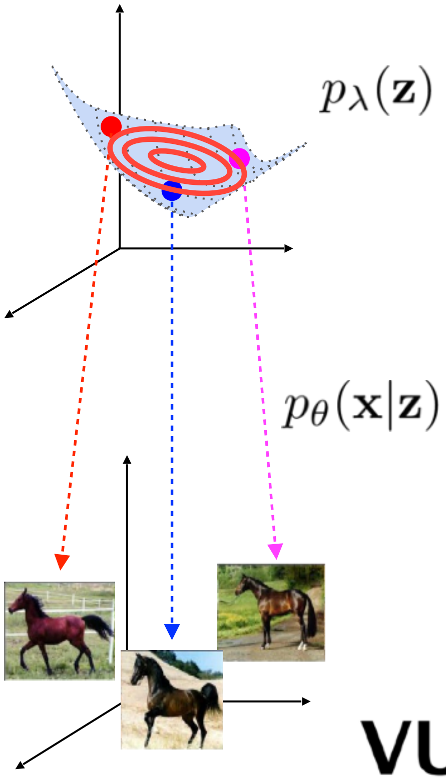

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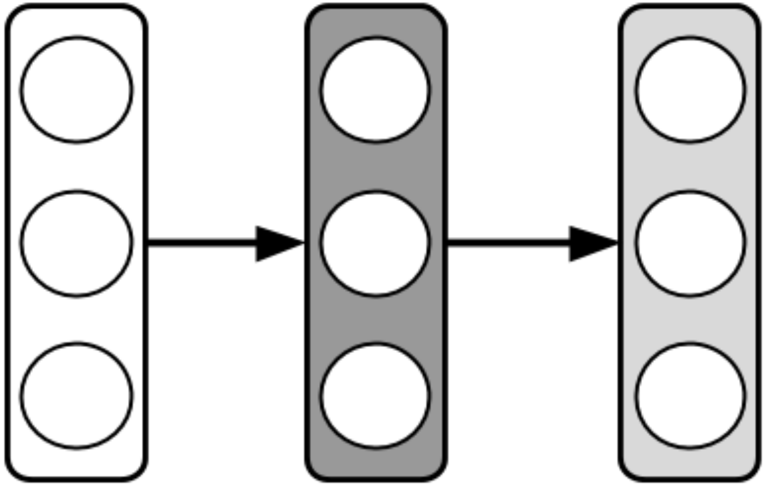
If we take standard Gaussian prior, we need to model p(xlz) only!

Let's consider a function *f* that transforms **z** to **x**.



$$\mathbf{z} \sim p_\lambda(\mathbf{z}) \qquad f(\mathbf{z}) \qquad \mathbf{x}$$

MacKay, D. J., & Gibbs, M. N. (1999). *Density networks*. Statistics and neural networks: advances at the interface. Oxford

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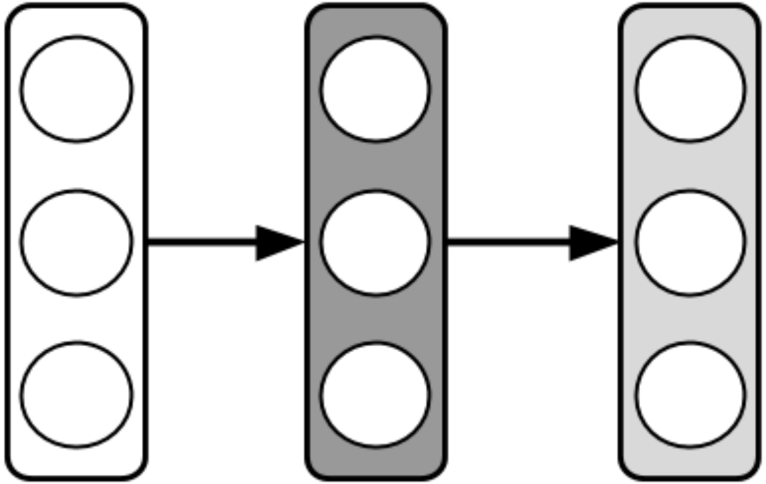

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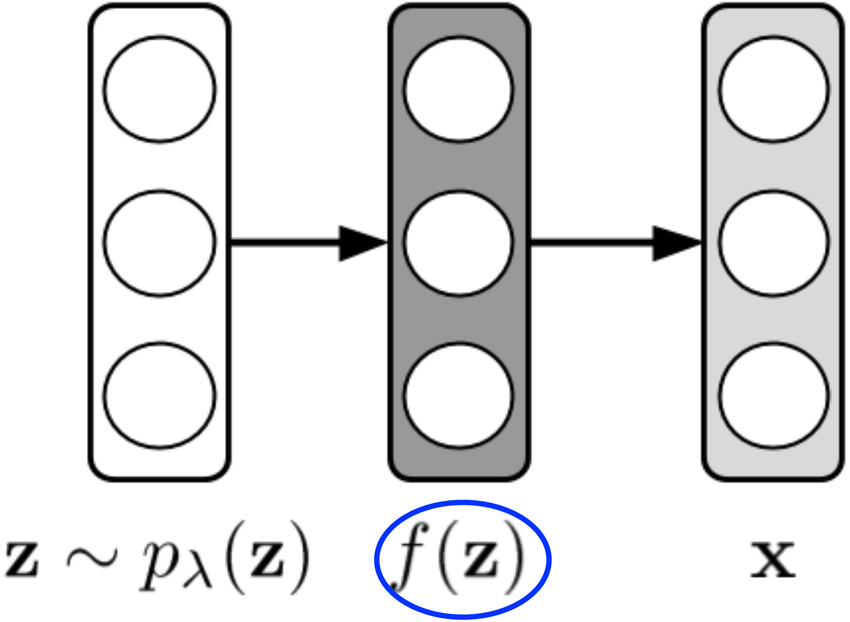

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NEURAL NETWORK 💪

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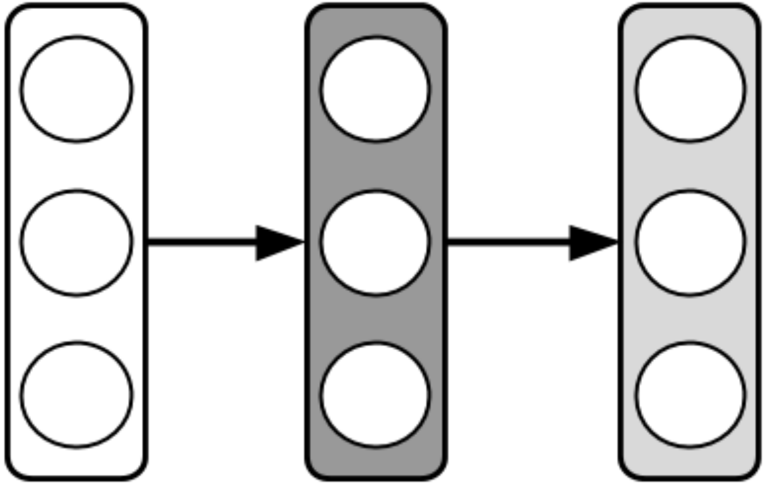

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Neural network outputs parameters of a distribution, e.g., a mixture of Gaussians.

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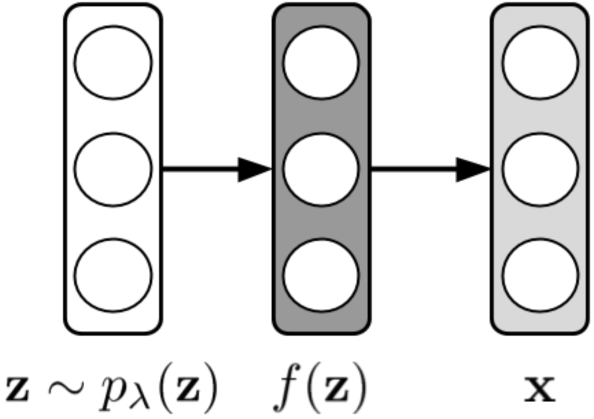

Training procedure:

1. Sample multiple **z**'s from the prior (e.g., standard Gaussian).

2. Apply log-sum-exp-trick, and apply backpropagation.

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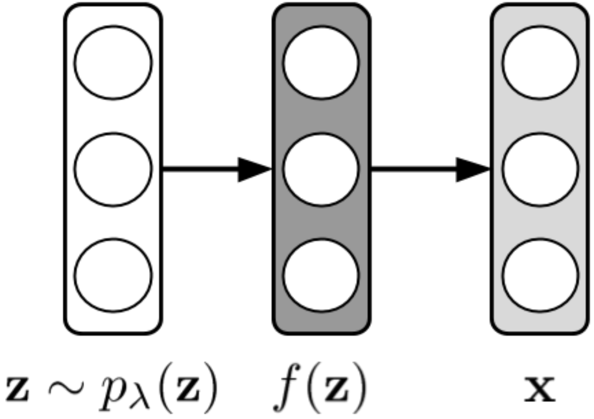

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Drawback: It scales badly in high-dimensional cases…

Advantages

✓ Non-linear transformations.

✓ Allows to generate.

Disadvantages

- No analytical solutions.
- No exact likelihood.
- It requires a lot of samples from the prior.
- Fails in high-dim.
- It requires an explicit distribution (e.g., Gaussian).

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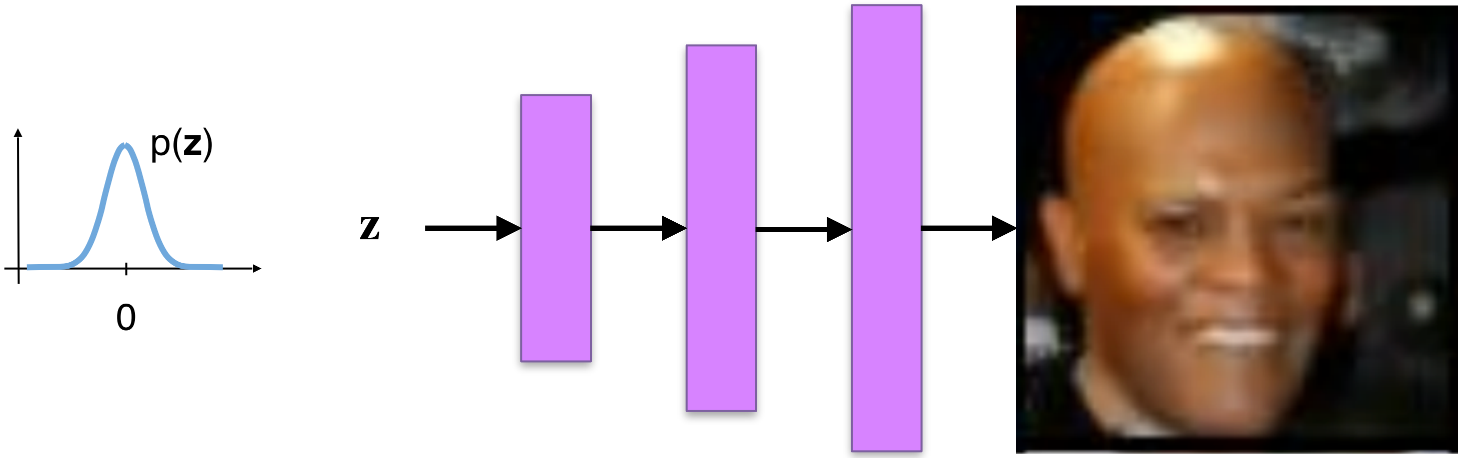
Can we do better?

IMPLICIT DISTRIBUTIONS

Let us look again at the Density Network model.

The idea is to inject noise to a neural network that serves as a **generator**:



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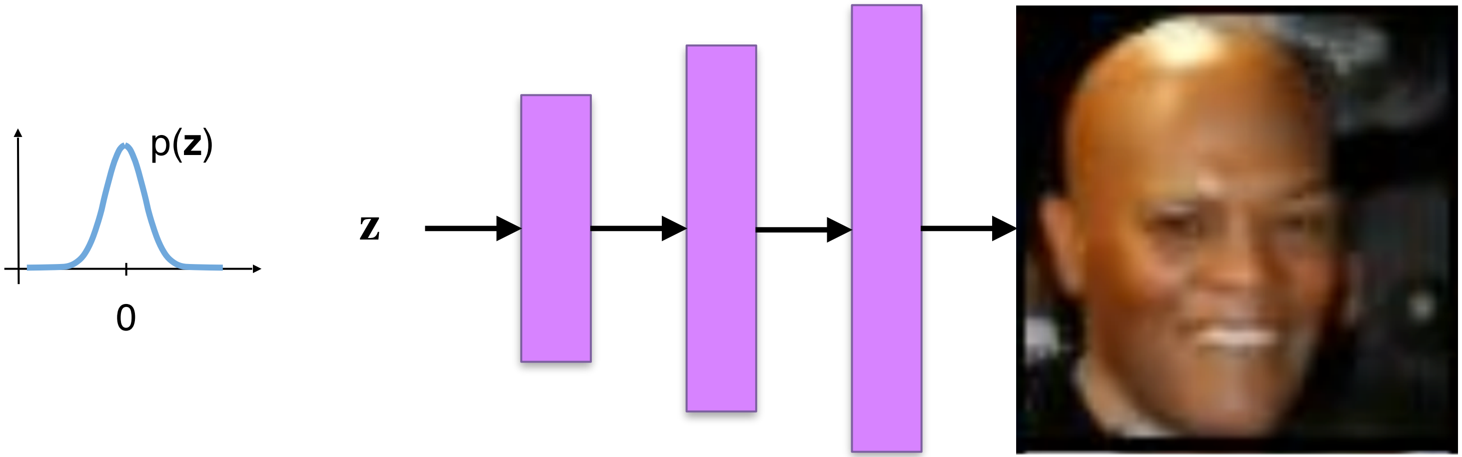

But now, we don't specify the distribution (e.g., MoG), but use a flexible transformation directly to return an image. This is now **implicit**.

THE IDEA BEHIND IMPLICIT DISTRIBUTIONS

We have a neural network (**generator**) that transforms noise into an image.

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We need to use a different approach.

GENERATIVE ADVERSARIAL NETWORKS

BEFORE WE GO INTO MATH…

Let’s imagine two actors:

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Let's imagine two actors:

A fraud

Let’s imagine two actors:

A fraud

An art expert

BEFORE WE GO INTO MATH…

Let’s imagine two actors:

A fraud

… and a real artist

An art expert

BEFORE WE GO INTO MATH…

Let’s imagine two actors:

The fraud aims to copy the real artist and cheat the art expert.

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… and a real artist

An art expert

The expert assesses a painting and gives her opinion.

Let's imagine two actors:

A fraud

The fraud aims to copy the real artist and cheat the art expert.

The fraud learns and tries to fool the expert.

An art expert

The expert assesses a painting and gives her opinion.

… and a real artist

Let’s imagine two actors:

A fraud

Hmmm…
A FAKE!

An art expert

… and a real artist

BEFORE WE GO INTO MATH…

Let’s imagine two actors:

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Let's imagine two actors:

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Hmmm…
PABLO!

An art expert

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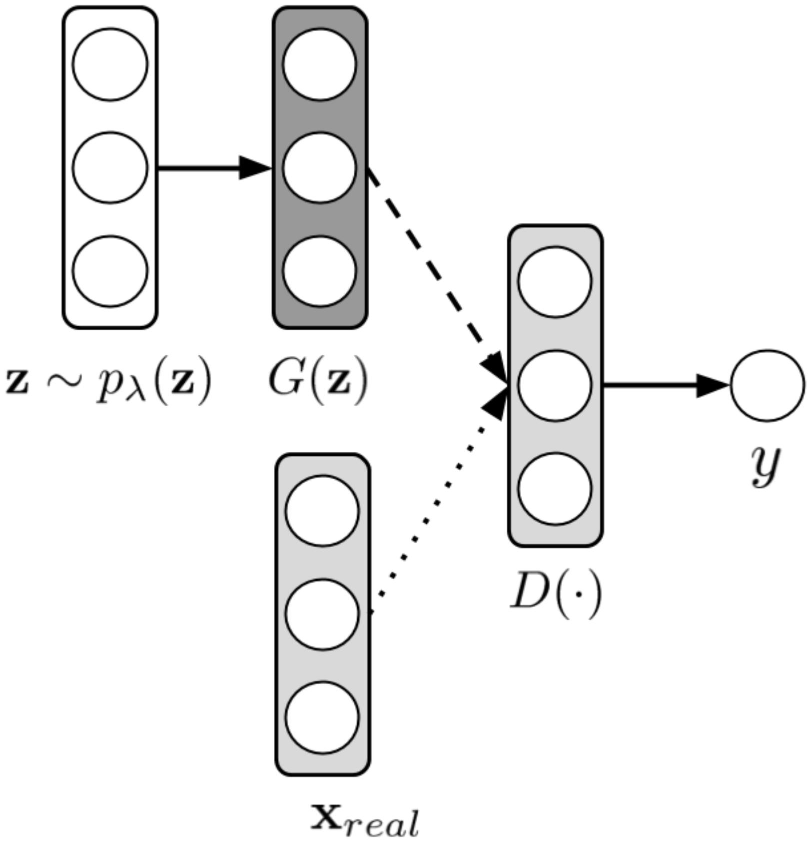
HOW WE CAN FORMULATE IT?



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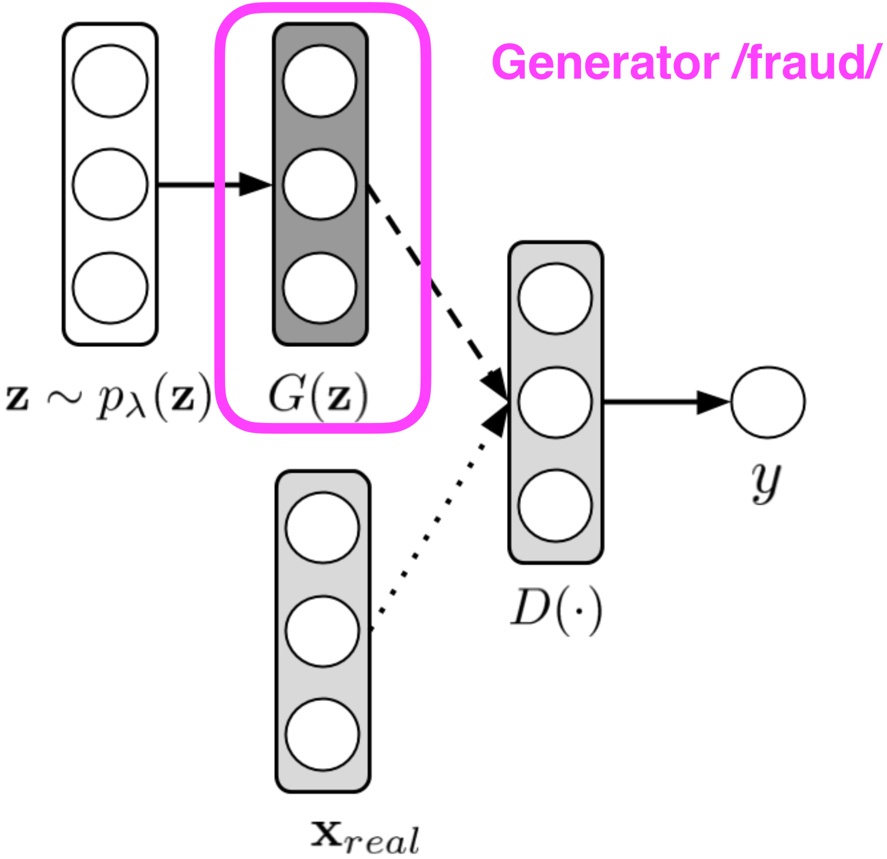

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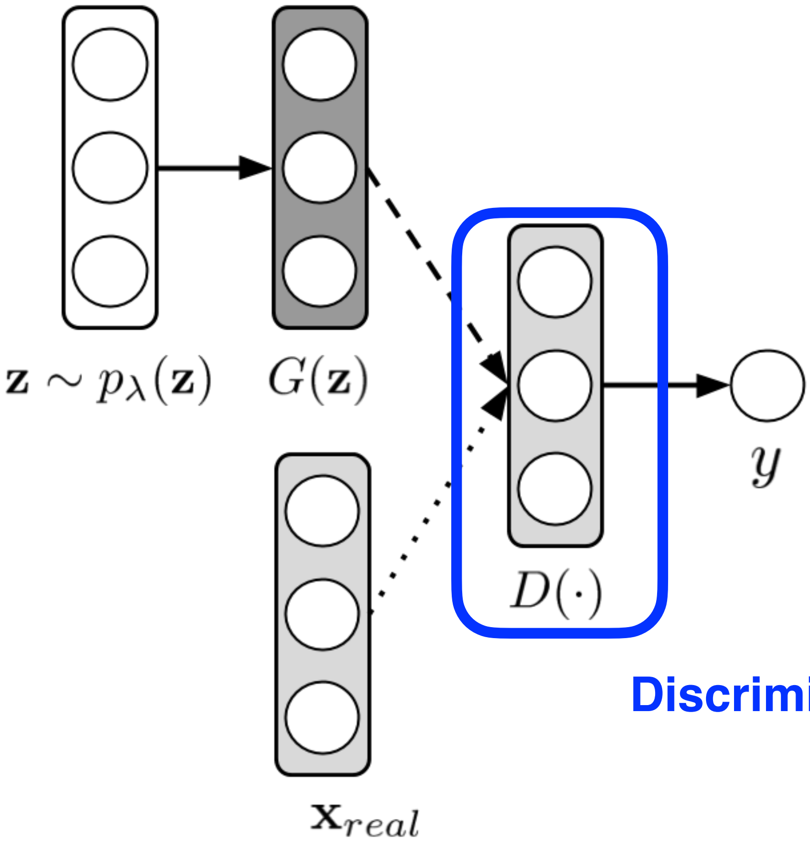


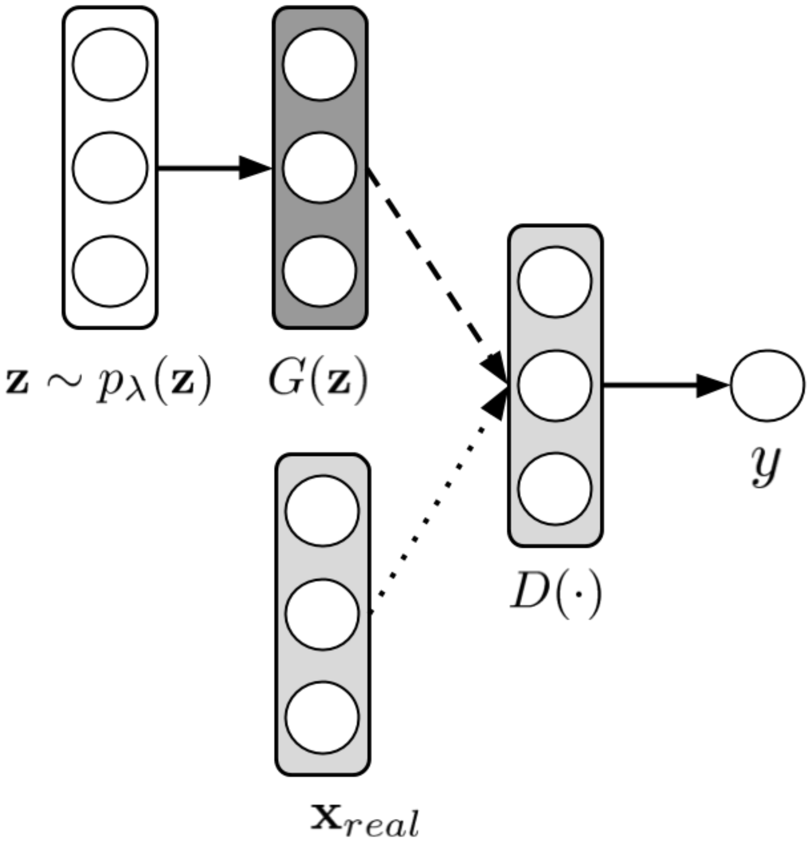
Discriminator /expert/

HOW WE CAN FORMULATE IT?

1. Sample **z**.

2. Generate *G*(**z**).

3. Discriminate whether given image is real or fake.



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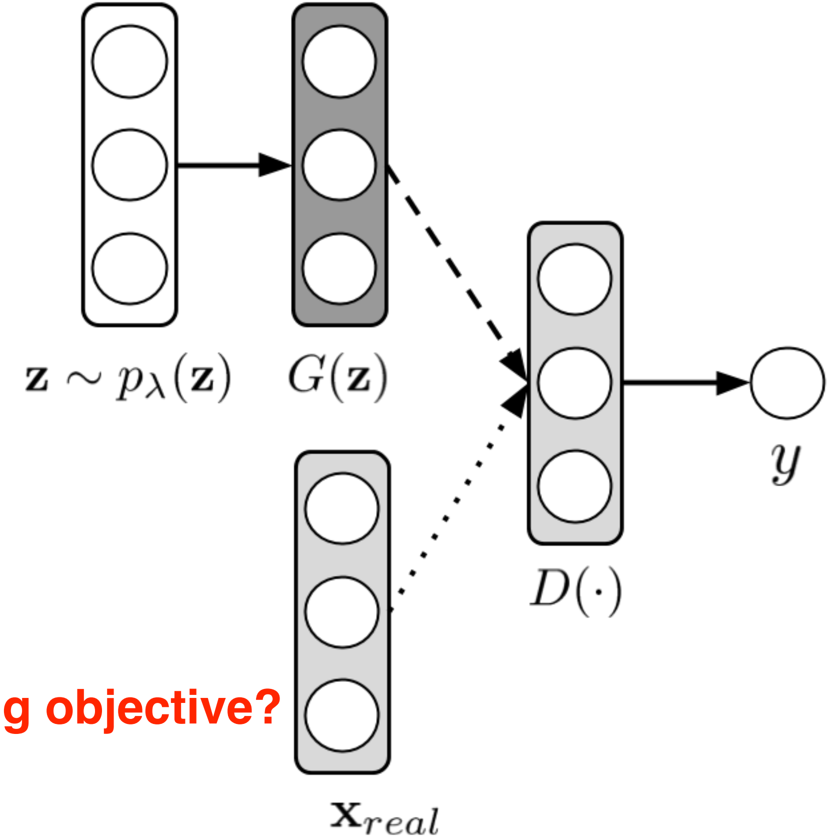

What about the learning objective?

ADVERSARIAL LOSS

We can consider the following learning objective:

$$\min_G \max_D \mathbb{E}_{\mathbf{x} \sim p_{real}}[\log D(\mathbf{x})] + \mathbb{E}_{\mathbf{z} \sim p(\mathbf{z})}[\log(1 - D(G(\mathbf{z})))]$$

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It resemblances the logarithm of the Bernoulli distribution:

$$y \log p(y = 1) + (1 - y)\log(1 - p(y = 1))$$

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Therefore, the discriminator network should end with a sigmoid function to mimic probability.

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Generative process:

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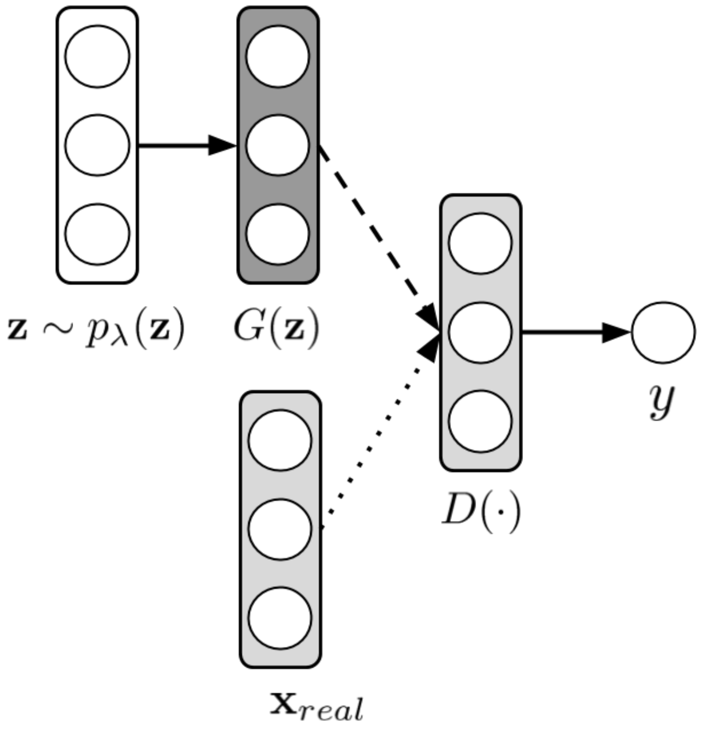

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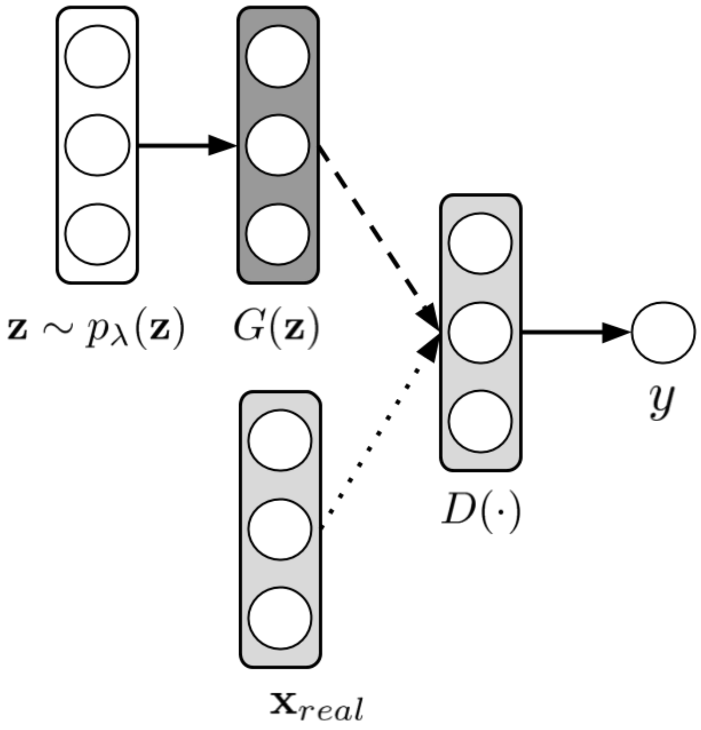

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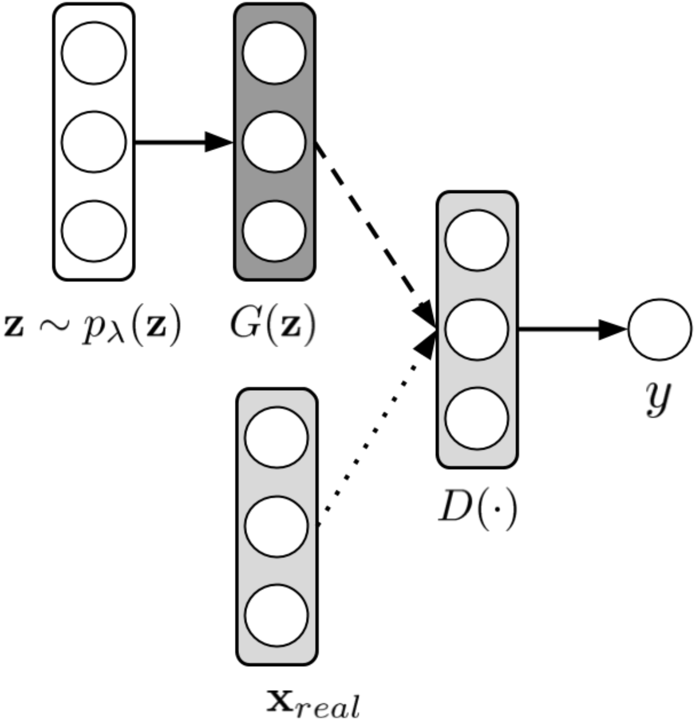

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Learning:

1. Generate fake images, and minimize wrt. *G*.

2. Take real & fake images, and maximize wrt. *D*.

GANS

```
import torch.nn as nn

class GAN(nn.Module):
    def __init__(self, D, M):
        super(GAN, self).__init__()
        self.D = D
        self.M = M

        self.gen1 = nn.Linear(in_features= self.M, out_features=300)
        self.gen2 = nn.Linear(in_features=300, out_features= self.D)

        self.dis1 = nn.Linear(in_features= self.D, out_features=300)
        self.dis2 = nn.Linear(in_features=300, out_features=1)
```

GANS

```
def generate(self, N):
    z = torch.randn(size=(N, self.D))
    x_gen = self.gen1(z)
    x_gen = nn.functional.relu(x_gen)
    x_gen = self.gen2(x_gen)
    return x_gen

def discriminate(self, x):
    y = self.dis1(x)
    y = nn.functional.relu(y)
    y = self.dis2(y)
    y = torch.sigmoid(y)
    return y
```

GANS

```
def gen_loss(self, d_gen):
    return torch.log(1. - d_gen)

def dis_loss(self, d_real, d_gen):
    # We maximize wrt. the discriminator, but optimizers minimize!
    # We need to include the negative sign!
    return -(torch.log(d_real) + torch.log(1. - d_gen))

def forward(self, x_real):
    x_gen = self.generate(N=x_real.shape[0])
    d_real = self.discriminate(x_real)
    d_gen = self.discriminate(x_gen)
    return d_real, d_gen
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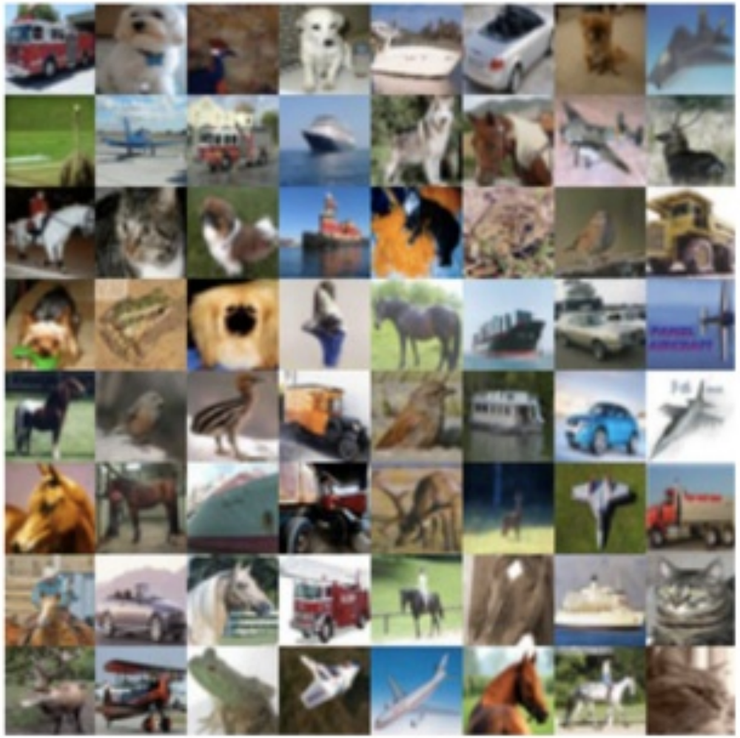
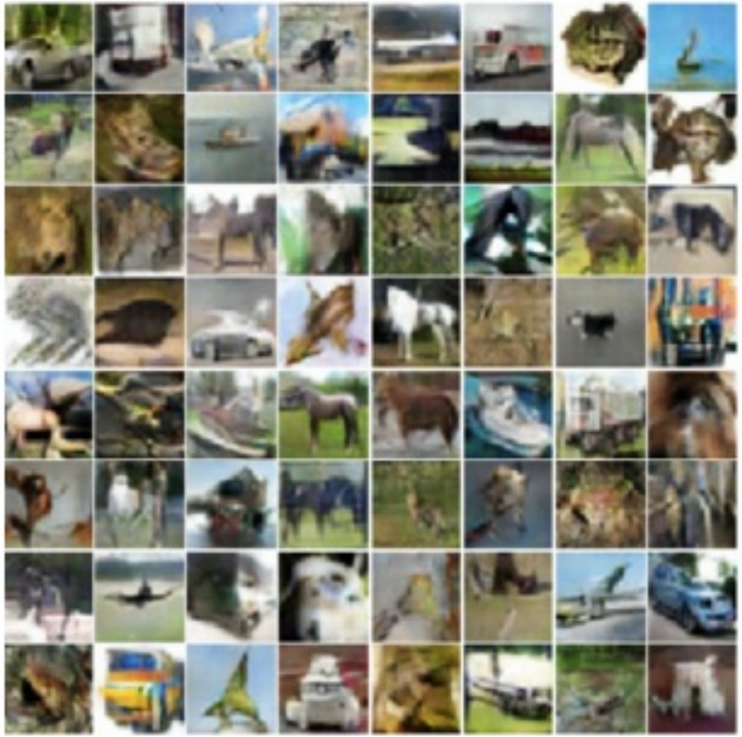
We can use two optimizers, one for d_real & d_gen, and one for d_gen.

GENERATIONS



Training Data



Samples

Salimans, T., et al. (2016). Improved techniques for training GANs. NeurIPS

GANS

Advantages

✓ Non-linear transformations.

✓ Allows to generate.

✓ Learnable loss.

✓ Allows implicit models.

✓ Works in high-dim.

Disadvantages

- No exact likelihood.
- **Unstable training**.
- *Missing mode problem* (i.e., it doesn't cover the whole space).
- No clear way for quantitative assessment.

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WASSERSTEIN GAN

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For instance, we can use the **earth-mover distance**:

$$\min_G \max_{D \in \mathscr{W}} \mathbb{E}_{\mathbf{x} \sim p_{real}}[D(\mathbf{x})] - \mathbb{E}_{\mathbf{z} \sim p(\mathbf{z})}[D(G(\mathbf{z}))]$$

where the discriminator is a 1-Lipschitz function.

Arjovsky, M., Chintala, S., & Bottou, L. (2017). Wasserstein GAN. arXiv preprint arXiv:1701.07875.

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It stabilizes training, but **other problems remain**.

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Thank you!

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